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An Absorbing Markov Chain for Accounts Receivables

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Abstract

This paper presents a case study that examines the nature and extent of timely payment of customer transactions on account of a Lean manufacturing organization. A discrete-time absorbing Markov chain model is applied as a tool for analyzing five transient states (new transaction, one-month overdue, two-months overdue, three months overdue, and four months overdue) and two competing absorbing states (paid in full and bad debt write-off). By employing a first-order Markov chain, transient states in the collections process are modeled by periods beginning with the completion of a new transaction. Ordinary Markov chain calculations are performed to determine the expected status of accounts receivables collections.

Keywords: Accounting, Markov chain, Transient states, Absorbing states, Computational analysis, Probability.

1 | Introduction

Given their relative simplicity, many systems can be successfully modeled by first-order Markov Chains to capture the necessary transitional and absorbing states adequately. A Markov chain is a stochastic process in which transitional states are random and, therefore, probabilistic.

Markovian models can be among the more useful and pragmatic to understand. Their broad application has been used extensively in the fields of reliability [1–4], software engineering [5–7], healthcare [7–9], telecommunications [10], education [11], [12], operations management (i.e., rework and scrapping [13], machine control with quality considerations [14], supply chain disruptions [15], plastic waste management systems [16], and inventory management [17], and accounting (i.e., repair and replacement [1], developing a

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model for estimating doubtful accounts from loss expectancy rates [18] and uncollectable debts [19] using steady-state transition probabilities). As a counter approach to using steady-state transition probabilities, Corcoran [20] generated forecasts of both accounts receivables collections and the inventory of next month's accounts receivables via an exponentially smoothed model of previous months' transition matrices. With the aid of Markov Chains, both Lewellyn and Johnson [21] and Stone [22] show that certain elements of accounts receivable, such as the number of day's sales outstanding and simple aging schedules, are influenced by monthly and seasonal sales variability. Randomness and uncertainty play important roles in modeling system behavior [23], [24].

In a Lean production environment, considerable emphasis is placed on concepts such as setup time reduction, waste and inventory reduction, work cells, and supply chain management regarding improving manufacturing and service systems. Critical Lean non-manufacturing systems, including accounting, human resources, and sales, are often overlooked, which may also dramatically impact a firm's bottom line. For example, Markov chain models can be employed to analyze accounts receivables collections, workforce planning, and sales forecasting. Non-manufacturing functions would benefit immensely from using this analytical tool concerning an organization's strategic plan, which is the motivation behind this paper.

A general discrete-time absorbing Markov chain model is presented to analyze the timeliness of payments on account of customer transactions at a manufacturing firm, a producer of corrugated boxes. The organization's chief financial officer obtained financial data regarding monthly revenues and collection probabilities and served to provide a computational example of the model's application.

This paper is organized in the following sections: Section 2 describes the manufacturing firm's credit policy. Section 3 provides a discussion of key assumptions and limitations. Section 4 introduces a mathematical background on Markov Chains. Section 5 provides technical aspects of Markov Chains. Section 6 discusses model definitions. Section 7 describes the goals of the study. Section 8 provides details on the computational analysis. Section 9 gives the results of the study. Section 10 presents a test for model adequacy with sensitivity analysis and addresses model uncertainty. Section 11 provides some concluding remarks. Finally, Section 12 offers model extensions and potential areas of future research.

2 | Credit Policy

To reduce the risk of bad debts, the manufacturer requires a minimum of three verifiable credit references in good standing before granting credit terms to new customers. Before a customer submits an order, the manufacturer's accounting department sends a blank credit application to the customer to be returned with company information, bank, and 3 trade references. All three trade references are contacted for a credit reference. A minimum of three good credit references is required to set up the potential customer on account with payment terms of net 30 days from the invoice date. If a new customer is denied credit because less than three references provided satisfactory credit referrals, the customer must make advanced payment in full of orders placed until satisfactory credit is obtained. This is achieved by paying all invoices on time for a period of six months with the manufacturer. Customers who fail to make payment when due are charged interest of one percent (1%) per month (twelve percent (12%) per annum. In its sole discretion, the manufacturer may revoke any credit extended to the customer and require payment in full before the next delivery of products. If the manufacturer retains a collection agency legal counsel or incurs any out-of-pocket expenses related to the collection of payments due from the customer, all such costs will be added to the sums due, will bear interest at the rate of 1% per month, and will be the responsibility of the customer.

3 | Key Assumptions and Limitations

Three key assumptions are: 1) the transition probabilities are in a constant, steady state rather than in a dynamic state from month to month or even within a month. Changes in transition probabilities over time may result as a function of changes in business activity, such as the use of sales promotions or increasing discount terms to stimulate sales, price increases to curb sales or reducing discount terms when the plant is

operating at peak capacity, and this may impact payment behavior, 2) by using historical data, partial payments are included in the historical transition and collection probabilities, and 3) the future is independent of the past, given the present. Similarly, the past is independent of the future, given the present.

A key limitation is that since the use of Markov Chains for accounts receivable depends on past observations of payment behavior, the question emerges regarding how practical previous collection percentages are if transition probabilities change over time, such as when experiencing higher than forecasted sales or declining economic conditions (i.e., pandemic, high inflation, supply chain disruptions, etc.)

4 | Mathematical Background on Markov Chains

Markov Chains use matrix algebra to forecast outcomes (states) from a given starting point and probabilities that describe the likelihood of transitioning from one state to another over time [25], [26]. A collection of random variables $\{X(0), X(1), X(2), \dots, X(n)\}$ is called a stochastic process.

Markov Chains have been commonly used to model stochastic processes that satisfy the following conditions:

I. The results must be one of a fixed number of states at each state.

II. If a system is in state i on one observation, then the conditional probability that it is in state j on the next observation depends only on i and j. This probability is denoted p_{ij}.

III. If a Markov chain consists of N states, then p_{ij} becomes the probability of making a direct transition from state i to state j; that is, $1 \leq i \leq N, 1 \leq j \leq N$.

$$P = [p_{ij}].$$

The matrix P is commonly known as the one-step transition matrix for the Markov chain.

If P is the one-step transition matrix for a Markov chain, then the matrix $P(k)$ of the k-step transition probabilities is $P(k) = P \times P \times P \times \cdots \times P = P^k$. The element of $P(k)$ that is in the *i*th row and *j*th column is denoted by $p_{ij}(k)$.

The Markov chain begins with a set of states, $S = \{s_1. s_2, s_3, \cdots, s_r\}$. The process begins in one of these states and successively moves from one state to another state with each move considered a step.

If we hypothesize that a Markov chain is currently in state s_i, then when the process moves from state s_i to s_j, it occurs with probability p_{ij}. However, if the process remains in its current state, it occurs with probability p_{ii}. An initial probability distribution, which specifies the starting state, is defined on S. The condition of the process at time $t + 1$ depends only on the condition of the process at time t such that $P = p_{ij} = P(x_{t+1} = j \mid x_t = 1)$, where $p_{ij} =$ the probability that given the system is in state i at time t, it will be in state j at time $t + 1$.

5 | Technical Background on Markov Chains

An absorbing state s_i of a Markov chain is called absorbing because once the process enters an absorbing state, it never leaves. Therefore, the state is an absorbing state when $p_{ii} = 1$ and all other $p_{ii} = 0$ for $i \neq j$. A Markov chain is absorbing if it has at least one absorbing state and if it is possible to go to an absorbing state from every state, although not necessarily in one step.

A non-absorbing state is called a transient in an absorbing Markov chain. State i is transient if there exist states j that are reachable from i, but state i is not reachable from any states j. Once the process leaves a transient state i, it will not return.

6 | Model Definitions

Our case study application presents a structured approach for analyzing the general likelihood of payment of customer transactions based on the present history of a corrugated box company's accounts receivables collections. Of primary interest are time to absorption and absorption probabilities by a transitional state.

An explanation of variables for both transition and absorption states follows. In this case study, customer transactions are characterized as existing in any one of five transitive states, as described in *Table 1*.

Table 1. Transient states.

Designation	Description
t_1	New transaction
t_2	Payment on transaction is one-month overdue
t_3	Payment on transaction is two-months overdue
t_4	Payment on transaction is three-months overdue
t_5	Payment on transaction is four-months overdue

Beginning in state t_1, a new customer transaction on account has been initiated for which payment is neither rendered nor currently due at the time of the transaction, as payment terms on account are typically net 30 days. State t_1, however, could progress into a subsequent state in which payment for a particular transaction eventually becomes overdue.

State t_2 is a condition in which a new transaction has surpassed the net 30-day payment due threshold and is now considered overdue. States t_3, t_4, and t_5 represent conditions whereby payment for a customer transaction has exceeded 60, 90, and 120 days, respectively, without payment.

Payment for customer transactions on account may migrate through all the transitive states in the payment process, eventually resulting in one of two absorbing states as described in *Table 2*.

Table 2. Absorbing states.

Designation	Description
a_1	Transaction is paid in full
a_2	Transaction is written off as a bad debt loss

Absorbing state a_1 indicates that a customer transaction on account is eventually paid in full. Conversely, absorbing state a_2 indicates that the transaction is eventually written off as a bad debt loss.

The data considered in this study consists of 2023 monthly sales for the manufacturer, as shown in *Table 3*.

Table 3. Monthly sales.

Month	Sales $
January	200,888
February	180,548
March	212,646
April	190,626
May	212,317
June	226,686
July	252,168
August	233,524
September	235,323
October	228,774
November	222,412
December	179,218

7 | Goals of the Study

A Markov model of accounts receivables was constructed to monitor the collection of individual customer order transactions for the organization. The goals of this study are as follows:

Goal #1: to determine the probability that a new transaction on account will be collected.

Goal #2: to determine the probability that a one-month overdue transaction will eventually become a bad debt.

Goal #3: to determine the probability that a two-month overdue transaction will eventually be paid in full.

Goal #4: to determine the probability that a three-month overdue transaction will eventually become a bad debt.

Goal #5: to determine the probability that a four-month overdue transaction will eventually be paid in full.

Goal #6: to estimate how much money is expected to go uncollected and, therefore, be written off as bad debt losses for the year.

8 | Computational Analysis

8.1 | Transition Matrix

If a Markov chain has s states, then the entry $p_{ij}(n)$ of the transition matrix P^n is the probability of being in state s_j after n steps, when the chain begins in state s_i such that P^n is of the canonical form

$$P_n = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}. \tag{1}$$

However, it should be noted that the transition matrix can have more than one canonical form.

In the transition matrix, the first t states are transient, and the last a states are absorbing. Q is a $t \times t$ matrix, R is a nonzero $t \times a$ matrix, 0 is an $a \times t$ matrix, and I is an $a \times a$ identity matrix.

The transition matrix is formed with transient states listed first, followed by absorbing states. Let s denote the total number of states in this study while m denotes the number of absorbing states $(a_1, a_2, \dots, a_m)$. It follows that $s - m$ denotes the number of transient states $(t_1, t_2, \dots, t_{n-m})$.

Past data indicate that the following Markov chain describes how the states of an account change from one month to the next month. Therefore, the transition matrix for the accounts receivables absorbing chain for the firm may be written in canonical form, as shown in *Table 4*.

Table 4. Transition matrix of transient states and absorbing states.

State		New	1 Month	2 Months	3 Months	4 Months	Paid in Full	Bad Debt
1	New	0	0.10	0	0	0	0.90	0
2	1 month	0	0	0.20	0	0	0.80	0
3	2 months	0	0	0	0.70	0	0.30	0
4	3 months	0	0	0	0	0.80	0.20	0
5	4 months	0	0	0	0	0	0.05	0.95
6	Paid in full	0	0	0	0	0	1	0
7	Bad debt	0	0	0	0	0	0	1

For example, there is a 10% chance that a new customer transaction will become overdue at the beginning of the next month. It then follows that there is a 20% chance that a one-month overdue transaction will evolve into a two-month overdue transaction at the beginning of the second month. In contrast, there is a 90% chance that a new customer transaction on account will be paid for in full within the first month. After four months, a customer transaction on account is absorbed into either paid in full status or written off as a bad debt loss.

8.2 | Transition Diagram

The transition diagram in *Fig. 1* aids in visualizing the interrelationships among the transient states and absorbing states. States 1 through 5 are transient states, and states 6 and 7 are absorbing states. Missing arrows indicate zero probability. The numbers above nodes 1-5 indicate probabilities of going from one transition

state to another. The numbers below nodes 1-5 indicate probabilities of going from a transition state to an absorbing state. Finally, the numbers for nodes 6 and 7 indicate that once a transaction enters an absorbing state, it never leaves.

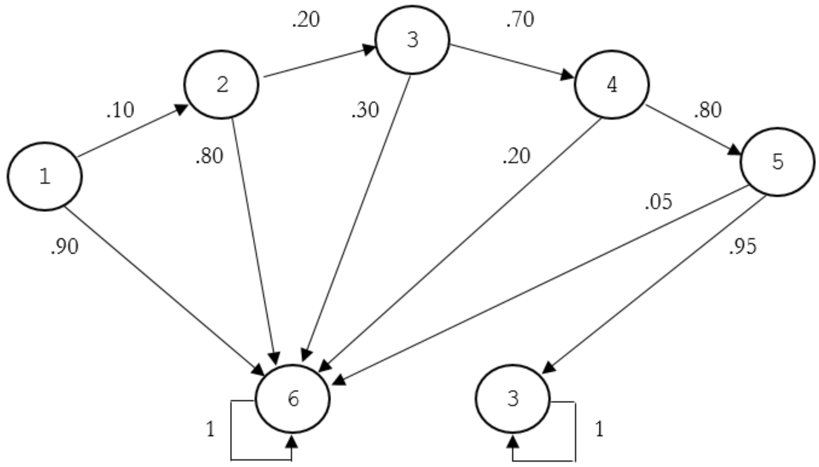

Fig. 1. Transition diagram.

Hence, $s = 5$, $m = 2$. The Q and R matrices below show transition state and absorbing state probabilities, respectively.

$$Q = \begin{bmatrix} 0 & 0.10 & 0 & 0 & 0 \\ 0 & 0 & 0.20 & 0 & 0 \\ 0 & 0 & 0 & 0.70 & 0 \\ 0 & 0 & 0 & 0 & 0.80 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \text{ and } R = \begin{bmatrix} 0.90 & 0 \\ 0.80 & 0 \\ 0.30 & 0 \\ 0.20 & 0 \\ 0.05 & 0.95 \end{bmatrix}. \quad (2)$$

The transition matrix of the firm's accounts receivables absorbing process is shown in *Eq. (3)*.

$$I - Q = \begin{bmatrix} 1 & -0.10 & 0 & 0 & 0 \\ 0 & 1 & -0.20 & 0 & 0 \\ 0 & 0 & 1 & -0.70 & 0 \\ 0 & 0 & 0 & 1 & -0.80 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (3)$$

8.3 | The Fundamental Matrix

For an absorbing Markov chain, the matrix $I - Q$ has an inverse $(I - Q)^{-1}$, also known as the fundamental matrix, used to find absorption probabilities, as shown in Table 6. The inverse matrix $(I - Q)^{-1}$ was obtained by applying the Gauss-Jordan method to the matrix $I - Q$ [27]. An absorbing Markov chain has at least one absorbing state, so every non-absorbing state will eventually transition into an absorbing state.

$$(I - Q)^{-1} = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{array} \begin{array}{c} \begin{array}{cccc} t_1 & t_2 & t_3 & t_4 \end{array} \\ \begin{bmatrix} 0.10 & 0.02 & 0.014 & 0.0112 \\ 1 & 0.20 & 0.14 & 0.112 \\ 0 & 1 & 0.70 & 0.56 \\ 0 & 0 & 1 & 0.80 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}. \quad (4)$$

8.4 | Time to Absorption

The ijth entry of the fundamental matrix $(I - Q)^{-1}R$ in *Eq. (4)* provides information about absorption probabilities regarding the stated goals of the firm. Here, the rows of $(I - Q)^{-1}R$ correspond to the transient states and the columns correspond to the absorbing states. The ijth element of $(I - Q)^{-1}R$ gives the probability that the chain which began in transient state t_i will be absorbed in the absorbing state a_j.

Therefore

$$(I - Q)^{-1}R = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{array} \begin{array}{c} \begin{array}{cc} a_1 & a_2 \end{array} \\ \begin{bmatrix} 0.99 & 0.01 \\ 0.98 & 0.11 \\ 0.47 & 0.53 \\ 0.24 & 0.76 \\ 0.05 & 0.95 \end{bmatrix} \end{array}. \quad (5)$$

9 | Results

A Markov accounts receivable model was constructed to monitor the collection of individual customer order transactions for the manufacturing firm. The revisited goals and corresponding results of the study are as follows:

Goal #1: to determine the probability that a new transaction on account will be collected. Result: the probability that a new transaction on account will be paid in full is element 11 (t_1 = new and a_1 = paid in full) of the matrix $(I - Q)^{-1}R$ = 0.99, or 99%.

Goal #2: to determine the probability that a one-month overdue transaction will eventually become a bad debt. Result: the probability that a one-month overdue transaction will eventually become a bad debt is revealed by element 22 (t_2 = one month overdue and a_2 = bad debt loss) of the matrix $(I - Q)^{-1}R$ = 0.11, or 11%.

Goal #3: to determine the probability that a two-month overdue transaction will eventually be paid in full. Result: The probability that a two-month overdue transaction will eventually be paid in full is revealed by element 31 (t_3 = two months overdue and a_1 = paid in full) of the matrix $(I - Q)^{-1}R$ = 0.47, or 47%.

Goal #4: to determine the probability that a three-month overdue transaction will eventually become a bad debt. Result: the probability that a three-month overdue transaction will eventually become a bad debt is revealed by element 42 (t_4 = three months overdue and a_2 = bad debt loss) of the matrix $(I - Q)^{-1}R$ = 0.76, or 76%.

Goal #5: to determine the probability that a four-month overdue transaction will eventually be paid in full. Result: the probability that a four-month overdue transaction will eventually be paid in full is revealed by element 51 (t_5 = four months overdue and a_1 = paid in full) of the matrix $(I - Q)^{-1}R$ = 0.05, or 5%.

Goal #6: to estimate how much money is expected to go uncollected and, therefore, be written off as bad debt losses for the year. Result: the probability that a new transaction will eventually become a bad debt is revealed by element 12 (t_1 = new and a_2 = bad debt loss) of the matrix $(I - Q)^{-1}R$ =0.01, or 1%. Since accounts receivables average \$214,619 per month, the amount of money that is estimated to go uncollected for the year is $0.01 \times (\$214{,}619 \times 12 \text{ months}) = \$25{,}754.$

Actual bad debt write-offs accounted for only \$7,406 of the firm's annual sales of \$2,575,428 (or~.29% of sales). While the actual bad debt loss is substantially less than the estimated bad debt of \$25,754 using the Markov model, both the firm's CEO and CFO attribute the nominal bad debt write-off to the implementation of a thorough credit reference policy. The administered credit policy is that the firm requires a minimum of three verifiable credit references in good standing before granting credit terms to a new customer. This rather extraordinary result reveals that the manufacturing firm's thorough credit review policy virtually assures payment full of customer transactions.

10 | Test for Model Adequacy

The Pearson Chi-Square Goodness of Fit Test is employed to test model adequacy. The following steps and hypothesis test demonstrate model adequacy for the Absorption Probabilities displayed in *Eq. 5*.

Table 5. Step 1: observed values.

	m_1	m_2	Totals
t_1	0.99	0.01	1.00
t_2	0.89	0.11	1.00
t_3	0.47	0.53	1.00
t_4	0.24	0.76	1.00
t_5	0.05	0.95	1.00
Totals	2.64	2.36	5.00

Table 6. Step 2: expected values.

	m_1	m_2
t_1	0.528	0.472
t_2	0.528	0.472
t_3	0.528	0.472
t_4	0.528	0.472
t_5	0.528	0.472

Table 7. Step 3: Chi-Square.

	m_1	m_2
t_1	0.40	0.45
t_2	0.25	0.28
t_3	0.01	0.01
t_4	0.16	0.18
t_5	0.43	0.48

Table 8 Step 4: test values.

X^2	2.65
d.f.	4
p-value (CHISQ.DIST.RT)	0.618799
p-value (CHITEST)	0.618799

10.1 | Pearson Chi-Square Goodness of Fit Test for Model Adequacy

Ho: $p_{11} = 0.99$, $p_{12} = 0.01$, …., $p_{51} = 0.05$, $p_{52} = 0.95$.

Ha: at least one p_i is not as specified in Ho.

$df = (5 - 1)(2 - 1) = 4.$

Rejection region ($\propto = 0.05$): Reject Ho if $\chi^2 > 9.488$.

Test Statistic: $\chi^2 = 2.65$.

Conclusion: Since $\chi^2 = 2.65 < 9.488$, we fail to reject Ho. The result is not statistically significant. We do not have enough evidence to conclude that Absorption Probabilities (pi's) distribution differs significantly from the expectations. Additionally, p-value = 0.6188 > $\propto = 0.05$ validates our decision not to reject Ho.

10.2 | Sensitivity Analysis

Sensitivity analysis is used to determine the rate of change in the output of a model (in this case, Paid in Full Absorption Probabilities) concerning changes in model inputs. Here, we want to determine the most influential inputs in the model.

From *Eq. 3*, by changing the Transition Matrix probabilities for each state by +/- increments of 5% over the range of -20% to +20% from the original t_i value for each state, while holding all other values constant, we can determine the effect on the Paid in Full Absorption Probability for a given t_i. The results of this sensitivity analysis are presented in *Table 9*.

Table 9. Changes in 'paid in full' absorption probabilities.

Change in t_i	New to 1 Month t_1 t_1 Value	% Paid in full	1 Month to 2 Months t_2 t_2 Value	% Paid in Full	2 Months to 3 Months t_3 t_3 Value	% Paid in Full	3 Months to 4 Months t_4 t_4 Value	% Paid in Full
-20%	-0.08	0.97	-0.16	0.87	-0.56	0.43	-0.64	0.23
-15%	-0.085	0.98	-0.17	0.88	-0.595	0.44	-0.68	0.23
-10%	-0.09	0.98	-0.18	0.88	-0.63	0.45	-0.72	0.24
-5%	-0.095	0.98	-0.19	0.89	-0.665	0.46	-0.76	0.24
0%	-0.1	0.99	-0.2	0.89	-0.7	0.47	-0.8	0.24
5%	-0.105	0.99	-0.21	0.90	-0.735	0.48	-0.84	0.24
10%	-0.11	1.00	-0.22	0.90	-0.77	0.48	-0.88	0.24
15%	-0.115	1.00	-0.23	0.91	-0.805	0.49	-0.92	0.25
20%	-0.12	1.00	-0.24	0.91	-0.84	0.50	-0.96	0.25

These changes are illustrated in *Fig. 2*. We observe that the steepest slope (i.e., most influential change) occurs for transition state t_3 within the original +/- 20% range; that is, going from payments that are 2 months overdue to 3 months overdue. Of all transition states, state t_3 is the most sensitive to changes in transition probabilities. We also note that the least sensitive transition state, but with very strong ramifications, occurs for transition state t_4 (orange line), indicating a strong likelihood that payments 3 months overdue tend to roll over into 4 months overdue and so on until either a collection agency or bad debt loss is decided upon to close the transaction.

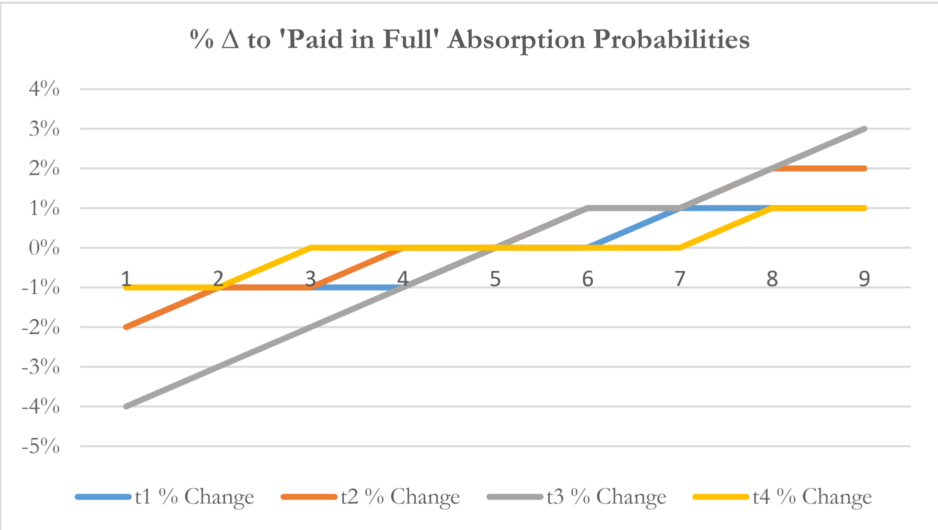

Fig. 2. % Changes to 'paid in full' absorption probabilities.

10.3 | Addressing Model Uncertainty

Uncertainty implies the absence of sureness in a phenomenon. Many possible reasons are attributed to model uncertainty. For example, uncertainty may exist in selecting input variables or model parameters. Initial conditions and boundary conditions impact model uncertainty. Other criteria impacting model uncertainty include assumptions, algorithms, objectives, and stopping criteria.

11 | Conclusion

The Lean philosophy emphasizes the efficient utilization of scarce resources. Accurate and prompt information is as much a controlled resource as machinery or people on the production floor. The dynamic use of information is often lost in myriad unnecessary reports. Markov chain models, however, can make efficient use of extracted information and enlighten management with decisive results that were previously unknown.

This case study has demonstrated the use of Markov chain analysis to compute a manufacturing firm's accounts receivable probability. By employing a first-order Markov chain, transient states in the collections process are modeled by periods beginning with the completion of a new transaction. The analysis consists of the expected time to absorption and absorption probability calculations. The results of this Markov chain study are consistent with those of Kallberg and Sanders [19] and Saibeni [26] in their work on accounts receivable collections.

Whereas sensitivity analysis reveals that transition state t_3 is most sensitive to percent changes in Transition Matrix probabilities, transition state t_4 is the least sensitive.

Equipped with the results obtained in this study, the firm's management can make more effective strategic decisions regarding the organization's financial future. For instance, the knowledge that 99% of all new customer transactions will eventually be paid in full enables management to prepare a cash flow analysis better, explore potential growth opportunities, and more accurately allocate their budget for future capital expenditures. Had this result been significantly lower, it would necessitate management's decision regarding tightening or relaxing credit policy restrictions. For example, management could tighten the credit granting requirement and enforce a more aggressive collection policy. Conversely, management could relax the same credit granting requirement to stimulate sales.

The interrelationships between the various transient states and corresponding absorbing states can be easily visualized and understood by modeling the firm's collections process using a Markov chain. Pure random fluctuations in transition probabilities over time may be larger for one firm than for another. An important point is to develop an analytical framework that organizes the detection and control of the collection experience and provides input information useful to the management decision process. Although more computationally extensive models may be explored, a firm that desires immediate and accurate results would benefit greatly by applying the principle of parsimony, reducing their model sophistication, analysis, and time to an appreciable yet concise level.

12 | Areas of Future Research

An extension of this Markov model for the manufacturing firm could be applied to employee workforce scheduling (i.e., a new employee stays a week, month, 6 months, 1 year, 5 years, over 5 years), customer complaints (i.e., whether a customer who complains once will complain again depending on the nature of the complaint), supply chain deliveries (i.e., early, on-time, late, missed deliveries), and evaluation of the extent to which new customers eventually result in long-term, repeat customers (i.e., orders weekly, monthly, quarterly, semi-annually, or annually).

Future research areas include applying Markov chains to model the probability that prospective customers who are contacted directly by a manufacturer's representative via cold calling will eventually buy a product for predicting the probability that potential blood donors will be accepted or rejected at a blood bank, and to model the probability that people who visit a new retail establishment will eventually become loyal customers.

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Conflicts of Interest

The authors declare no conflict of interest.

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